Vector Concepts

In This Module

- We are going to look at the question: "What is a vector?"

NASA. [Photo]. STS-132 Shuttle Mission Imagery. Retrieved from http://spaceflight.nasa.gov/gallery/images/shuttle/sts-132/html/sts132-s-074.htm

In This Unit

- We will learn how vectors can be used to represent many different quantities in the real world.
- We will solve related problems.

Language

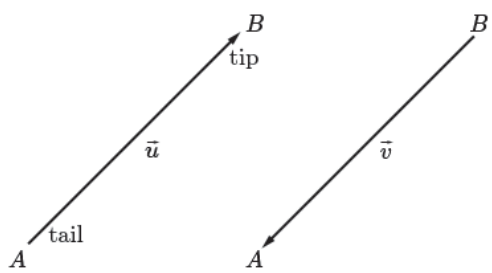
A **scalar** is a quantity with magnitude only.

A **vector** is a quantity with both magnitude **and** direction.

Scalar	Vector
Distance is a scalar quantity e.g., Keri lives 6 km from the University of Waterloo	**Displacement** is distance in a given direction e.g., Keri lives 6 km northeast of the University of Waterloo
Speed is a scalar e.g., the plane is travelling at a speed of 852 km/h	**Velocity** is speed in a given direction e.g., the velocity of the plane is 852 km/h west
Mass is a scalar e.g., Mr. Ford has a mass of 100 kg	**Weight** is force downwards due to gravity e.g., Mr. Ford has a weight of 980 N (downwards)

Representation

To represent vectors we use **rays** (directed line segments).



$\overrightarrow{AB} = 200$ km/h NE $\qquad$ $\overrightarrow{BA} = 200$ km/h SW

Vectors can also be labelled using single lowercase letters, e.g., $\vec{u} = 200$ km/h NE and $\vec{v} = 200$ km/h SW.

The length of the ray is a positive real number, which represents the **magnitude** of the vector.

If the magnitude of a vector is zero, we call it the **zero vector** and denote it $\vec{0}$. This is a useful vector despite that its direction is undefined.

Magnitude

$$\vec{u} = 200 \text{ km/h NE and } \vec{v} = 200 \text{ km/h SW}$$

To show magnitude, we place the vector name in absolute value bars, e.g., $|\vec{u}| = 200$ km/h.

Opposite vectors have the same magnitude but are opposite in direction; e.g., from the preceding example, $\vec{u} = -\vec{v}$ but $|\vec{u}| = |\vec{v}|$.

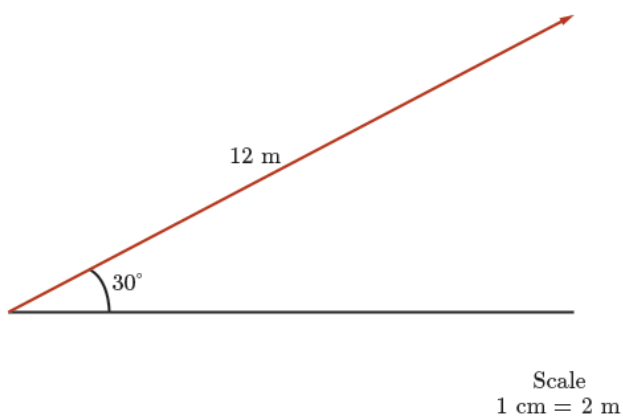
Two vectors are **equal** if and only if they have the same **magnitude** and **direction**. They do not need to be in the same position.

Examples

Example 1

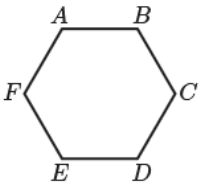
Draw a vector to represent the displacement of a box moved 12 m up a ramp which is inclined at 30° to the horizontal.

Solution



Examples

Example 2

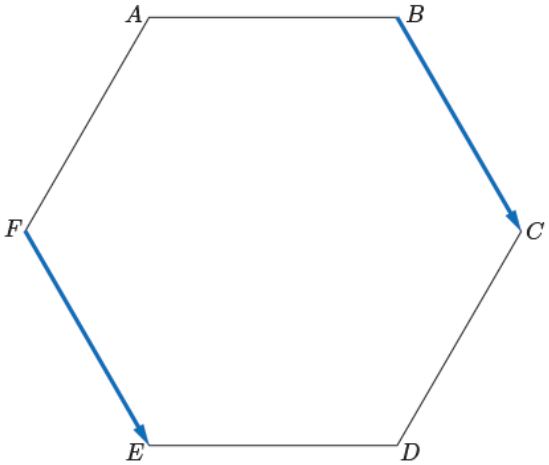


$ABCDEF$ is a regular hexagon. Give examples of vectors formed between pairs of vertices of hexagon $ABCDEF$:

a. equal

Solution

$\overrightarrow{AB} = \overrightarrow{ED}$
$\overrightarrow{FE} = \overrightarrow{BC}$



Examples

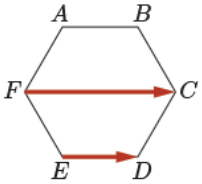
Example 2

$ABCDEF$ is a regular hexagon. Give examples of vectors formed between pairs of vertices of hexagon $ABCDEF$:

Solutions

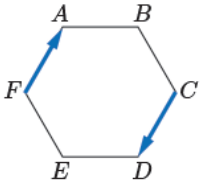
b. parallel but with different magnitudes

Possible Solution: $\overrightarrow{ED}$ and $\overrightarrow{FC}$



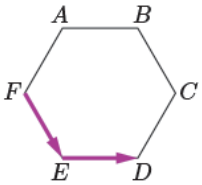
c. equal in magnitude but opposite in direction

Possible Solution: $\overrightarrow{FA}$ and $\overrightarrow{CD}$



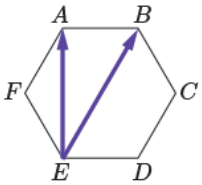
d. equal in magnitude but not parallel

Possible Solution: $\overrightarrow{FE}$ and $\overrightarrow{ED}$



e. different in both magnitude and direction

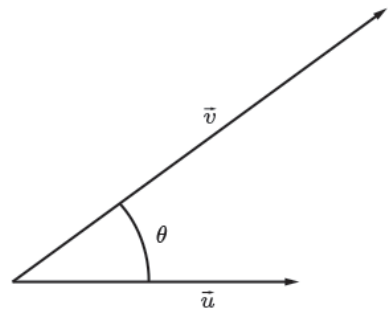
Possible Solution: $\overrightarrow{EA}$ and $\overrightarrow{EB}$



Vector Direction

To specify direction, we state a compass direction or the angle it makes with the horizontal or vertical (e.g., 30° above the horizontal or N 20° W).

We can also state its direction relative to another vector.



Angle Between Vectors

The angle between two vectors is the angle $\leq 180^\circ$ formed when the vectors are placed **tail to tail**, that is, starting at the same point.

Examples

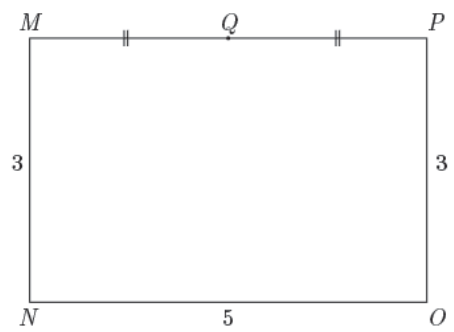
Example 3

$MNOP$ is a rectangle with side lengths 3 and 5.

Q is the midpoint of MP.

Find the angle between the following vectors:

a. $\overrightarrow{NP}$ and $\overrightarrow{NO}$



Examples

Example 3

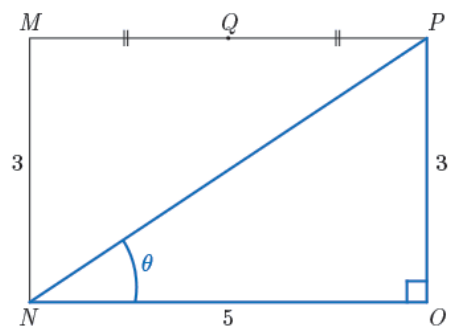
$MNOP$ is a rectangle with side lengths 3 and 5.
Q is the midpoint of MP.
Find the angle between the following vectors:
a. $\overrightarrow{NP}$ and $\overrightarrow{NO}$

Solution

$\tan\theta = \frac{3}{5}$

$\theta = \tan^{-1}\left(\frac{3}{5}\right)$

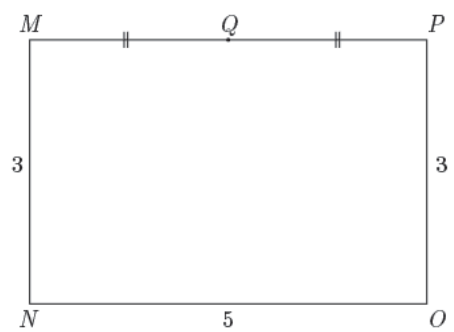
$\theta \approx 30.96^\circ$



Examples

Example 3

$MNOP$ is a rectangle with side lengths 3 and 5.
Q is the midpoint of MP.
Find the angle between the following vectors:
b. $\overrightarrow{NM}$ and $\overrightarrow{NQ}$



Examples

Example 3

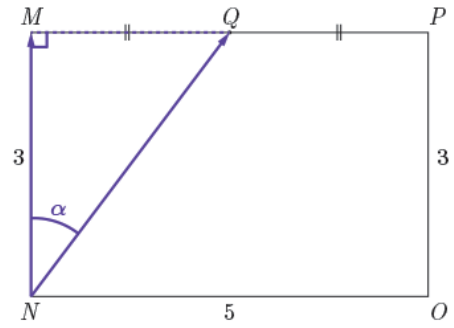
$MNOP$ is a rectangle with side lengths 3 and 5.
Q is the midpoint of MP.
Find the angle between the following vectors:
b. $\overrightarrow{NM}$ and $\overrightarrow{NQ}$

Solution

$\tan\alpha = \frac{2.5}{3}$

$\alpha = \tan^{-1}\left(\frac{2.5}{3}\right)$

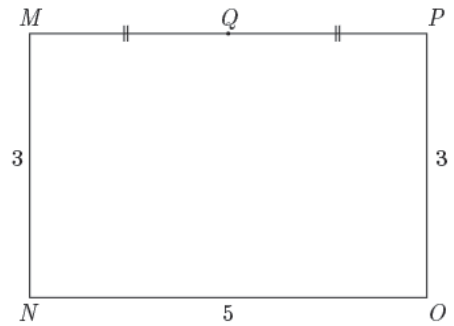
$\alpha \approx 39.81^\circ$



Examples

Example 3

$MNOP$ is a rectangle with side lengths 3 and 5.
Q is the midpoint of MP.
Find the angle between the following vectors:
c. $\overrightarrow{NQ}$ and $\overrightarrow{QP}$



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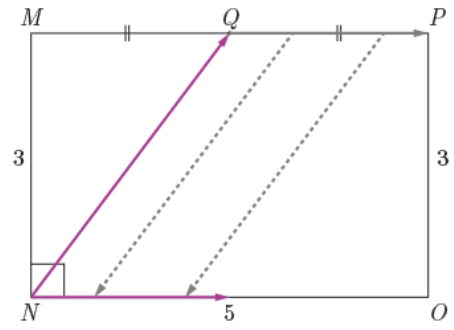


Examples

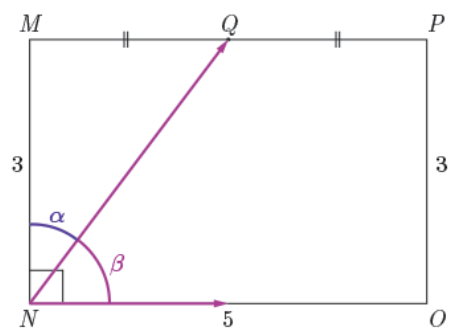
Example 3

$MNOP$ is a rectangle with side lengths 3 and 5.
Q is the midpoint of MP.
Find the angle between the following vectors:
c. $\overrightarrow{NQ}$ and $\overrightarrow{QP}$

$\beta = 90^\circ - \alpha$

$\beta \approx 90^\circ - 39.81^\circ$

$\beta \approx 50.19^\circ$

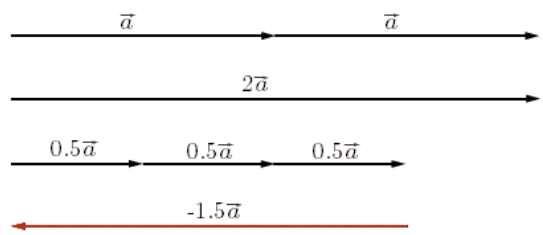


Scalar Multiplication

When two vectors are parallel, one of the vectors can be expressed in terms of the other using **scalar multiplication**.

In general, given some real number k, $k\vec{a}$ is a vector with the following attributes:

i. Its magnitude is $|k\vec{a}| = |k||\vec{a}|$.
ii. Its direction is the same as $\vec{a}$ if $k > 0$ and opposite to $\vec{a}$ if $k < 0$ (if $k = 0$, then $k\vec{a} = \vec{0}$).



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Example 4

Given that $|\vec{v}| = 5$, find the magnitude of each of the following vectors:

a. $2\vec{v}$

Solution

$|2\vec{v}| = |2||\vec{v}| = (2)(5) = 10$

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b. $-4\vec{v}$

c. $\frac{1}{5}\vec{v}$

Solution

b. $|-4\vec{v}| = |-4||\vec{v}| = (4)(5) = 20$

c. $\left|\frac{1}{5}\vec{v}\right| = \left|\frac{1}{5}\right||\vec{v}| = \frac{1}{5}(5) = 1$

The vector $\frac{1}{5}\vec{v}$ is a special vector called a **unit vector** because it has magnitude 1.

Unit Vector

To create a unit vector in the direction of a non-zero $\vec{u}$, multiply $\vec{u}$ by the scalar equal to the reciprocal of the magnitude of $\vec{u}$.

That is, $\vec{v} = \frac{1}{|\vec{u}|}\vec{u}$ is a unit vector in the direction of $\vec{u}$.

Notationally, this is written $\hat{u} = \frac{1}{|\vec{u}|}\vec{u}$.

Example 5

If $|\vec{a}| = 12$, state a unit vector in the direction of $\vec{a}$.

Solution

A unit vector in the direction of $\vec{a}$ would be the vector $\vec{a}$ multiplied by a scalar equal to the reciprocal of $|\vec{a}|$. That is,

$$\hat{a} = \frac{1}{12}\vec{a}$$

Conclusion

In this module, we have

- distinguished the difference between a **vector** and a **scalar**,
- defined **opposite vectors**, **equal vectors**, and the **magnitude of a vector**,
- looked at how to determine the angle between two vectors,
- talked about **scalar multiplication**, and
- talked about a **unit vector**.