The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

Solving Polynomial Equations And Inequalities Using Technology

Introduction

Polynomial functions can be used to model many real-world situations from revenue and cost formulas in business, to population predictions in science, to bridge designs in engineering.

It is very unlikely, when working with the polynomial functions in these circumstances, that the equations can be solved easily using the factoring techniques taught in this unit.

Although there are other algebraic methods for solving these polynomial equations, such as the Bisection method or Newton's method (which requires calculus), solutions to these equations can be determined efficiently using graphing technology.

You may wish to research the algebraic approaches mentioned. However, study of these concepts are covered in post-secondary mathematics.

In this module, we will demonstrate the use of graphing technology, by utilizing the Maple worksheet provided, to solve a real-life problem involving polynomial equations and inequalities.

Example

Problem

The annual revenue, for a small start up company, is modeled by the function

$$R(t) = t^3 - 10.5t^2 + 30t + 14, t \geq 0$$

where R is the revenue in tens of thousands and t is the number of years since the company was first formed in 2000.

a. What was the revenue of the company the year it was established?

b. In which year, or years, will the annual revenue of the company be $\$350\,000$?

c. The company chose to expand when the annual revenue was greater than $\$500\,000$. In which year did this expansion occur?

d. The annual cost to run the company can be modeled by $C(t) = -0.85t + 35.8$, where C is the cost in tens of thousands and t is the time in years since 2000. During which year, or years, was the company not making a profit?

e. Are there any real-world limitations to this model?

Example

Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

a. What was the revenue of the company the year it was established?

Solution

The company was founded in 2000, so $t = 0$.

$$R(0) = (0)^3 - 10.5(0)^2 + 30(0) + 14$$
$$R(0) = 14$$

Therefore, the revenue generated in 2000 was $\$140\,000$ (since R is revenue in tens of thousands, revenue $= R \times 10000$).

Example

Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

b. In which year, or years, will the annual revenue of the company be $\$350\,000$?

Solution

We need to find t, when $R(t) = 35$.
(Remember R is the revenue in tens of thousands, so $R = 350000 \div 10000$.)

We need to solve

$$35 = t^3 - 10.5t^2 + 30t + 14$$

Using graphing technology, we can do this one of two ways.

Example

Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

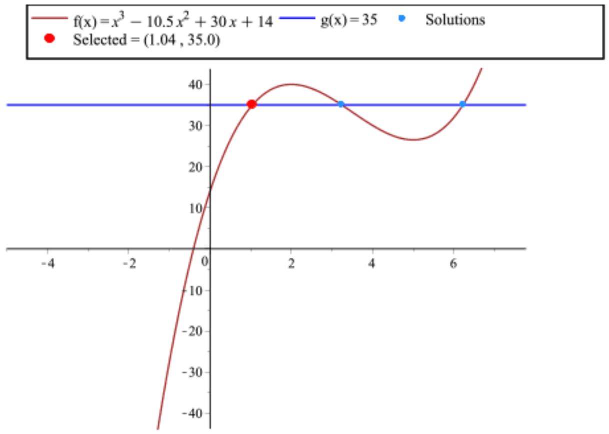
b. In which year, or years, will the annual revenue of the company be $\$350\,000$?

Solution Method 1

We can graph $f(x) = x^3 - 10.5x^2 + 30x + 14$ and $g(x) = 35$ and find the points of intersection of the two graphs.

The two functions have been entered into the Maple worksheet.

By selecting a point of intersection, its coordinates will appear on the screen.



We can see here that the revenue is $\$350\,000$ (i.e., $R = 35$) when $x = 1.04$. By selecting the other two points of intersection, we find $x = 3.24$, and $x = 6.22$.

Therefore, the revenue of the company will be $\$350\,000$ at some point in the years 2001, 2003, and 2006.

Example

Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

b. In which year, or years, will the annual revenue of the company be $\$350\,000$?

Solution Method 2

To solve $35 = t^3 - 10.5t^2 + 30t + 14$, we rearrange the equation.

$$0 = t^3 - 10.5t^2 + 30t - 21$$

We can now graph

$$f(x) = x^3 - 10.5x^2 + 30x - 21$$

using the Maple worksheet and find the x-intercepts.

Example

Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14, \ t \geq 0$$

b. In which year, or years, will the annual revenue of the company be $\$350\,000$?

Solution Method 2

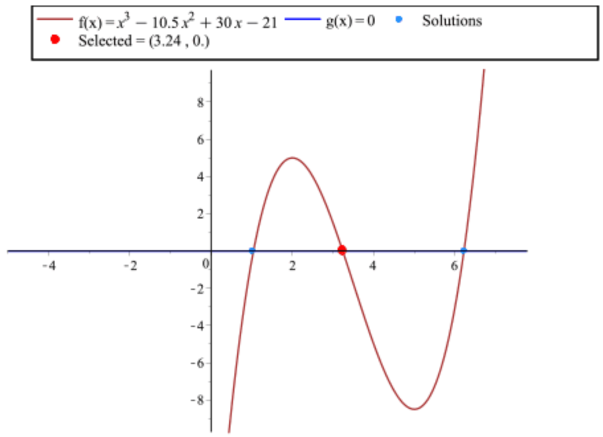
We enter the function

$$f(x) = x^3 - 10.5x^2 + 30x - 21$$

into the Maple worksheet.

By selecting an x-intercept on the graph, its coordinates will be identified (note the software assumes $g(x) = 0$).
Again, $x = 1.04, 3.24$, and 6.22.

Therefore, the revenue of the company will be $\$350\,000$ at some point in the years 2001, 2003, and 2006.



Example

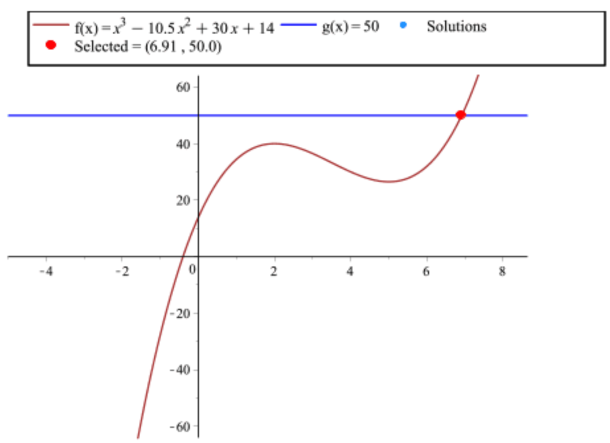
Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14, \ t \geq 0$$

c. The company chose to expand when the annual revenue was greater than $\$500\,000$. In which year did this expansion occur?

Solution

We need to determine when $R(t) > 50$.
Replacing $R(t)$ with $f(x)$, we will graph $f(x) = t^3 - 10.5t^2 + 30t + 14$ and $g(x) = 50$ and determine when $f(x) > g(x)$.
In the graph, $f(x) = g(x)$ when $x = 6.91$.
This can be interpreted to mean that the company will choose to expand near the end of 2006.



Example

Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

c. The company chose to expand when the annual revenue was greater than $500 000. In which year did this expansion occur?

Solution

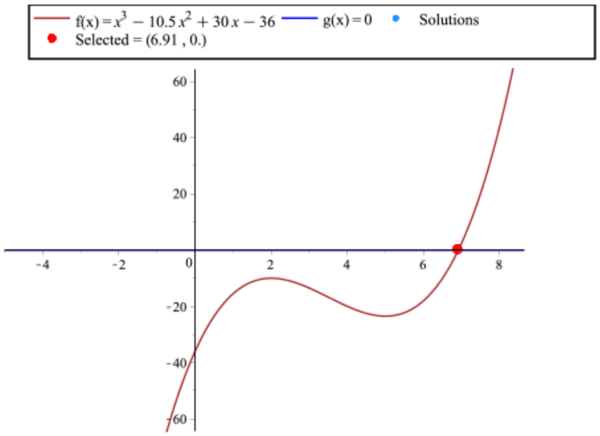
Another approach would be to rearrange the equation algebraically

$$t^3 - 10.5t^2 + 30t + 14 > 50$$
$$t^3 - 10.5t^2 + 30t - 36 > 0$$

then graph

$$f(x) = x^3 - 10.5x^2 + 30x - 36$$

and determine when $f(x) > 0$ (above the x-axis).



Example

Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

d. The annual cost to run the company can be modeled by $C(t) = -0.85t + 35.8$, where C is the cost in tens of thousands and t is the time in years since 2000. During which year, or years, was the company not making a profit?

Solution

$$\text{Profit} = \text{Revenue - Cost}$$
$$P(t) = (t^3 - 10.5t^2 + 30t + 14) - (-0.85t + 35.8)$$
$$P(t) = t^3 - 10.5t^2 + 30.85t - 21.8$$

We need to determine when $P(t) > 0$, by graphing the profit function

$$f(x) = x^3 - 10.5x^2 + 30.85x - 21.8$$

and determining when $f(x) > 0$.

Example

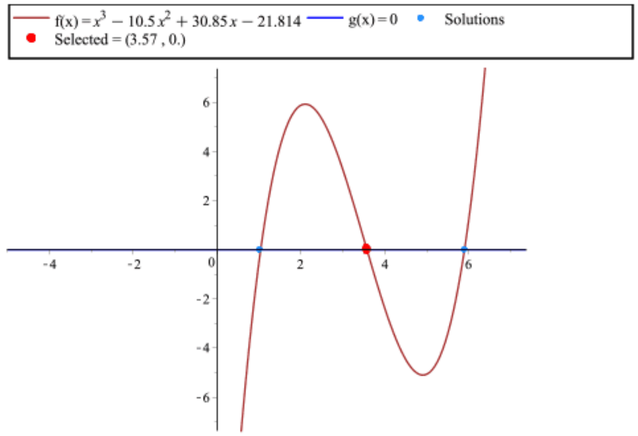
Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

d. The annual cost to run the company can be modeled by $C(t) = -0.85t + 35.8$, where C is the cost in tens of thousands and t is the time in years since 2000. During which year, or years, was the company not making a profit?

Solution

The x-intercepts occur at $x = 1.04,\ 3.57$, and 5.89.
This can be interpreted to mean that the company was not making any profit from 2000 to 2001, and between July 2003 and November 2005.



Example

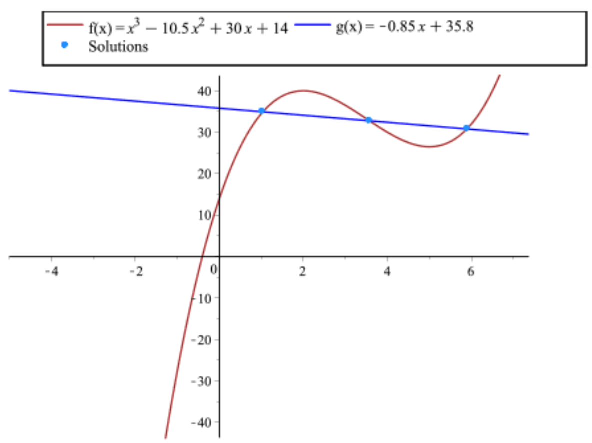
Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

d. The annual cost to run the company can be modeled by $C(t) = -0.85t + 35.8$, where C is the cost in tens of thousands and t is the time in years since 2000. During which year, or years, was the company not making a profit?

Solution

Another approach would be to graph the revenue and cost functions, $f(x) = x^3 - 10.5x^2 + 30x + 14$ and $g(x) = -0.85x + 35.8$, and determine when $f(x) > g(x)$.



Example

Problem

$$R(t) = t^3 - 10.5t^2 + 30t + 14,\ t \geq 0$$

e. Are there any real-world limitations to this model?

The equations may model the revenue and/or costs for the company for a limited period of time.

With this equation, as time (t) increases in value, the revenue and profit will also increase without bound, which is highly unlikely.

Also, cost decreases over time and will be approximately equal to zero around 2042. This will not happen in a real-world situation.