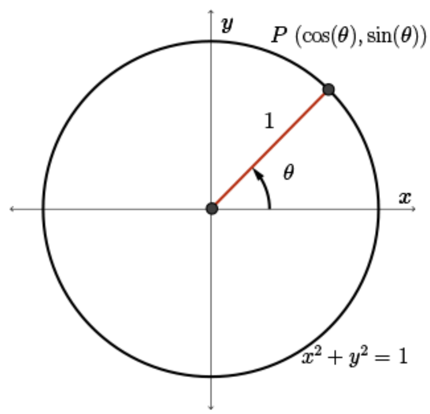
The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

Graphs of Primary Trigonometric Functions

Introduction

In other modules, the unit circle was introduced and used to determine the exact trigonometric ratios for multiples of $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$ radians and multiples of $30^\circ, 45^\circ, 60^\circ$, and 90°.

We discovered that any point, P, on the unit circle and also on the terminal arm of a standard position angle, θ, can be written $P\left(\cos(\theta), \sin(\theta)\right)$.



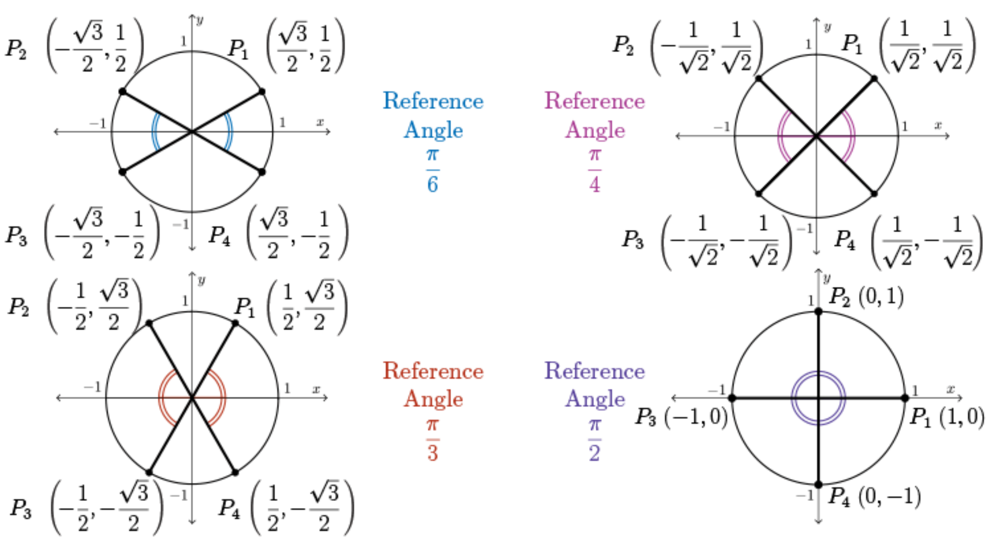
We will use this background to help us draw the graphs for the three primary trigonometric functions.
New terminology will be introduced as we examine properties associated with these graphs.

Primary Trigonometric Functions:

$$y = \sin(\theta),\ y = \cos(\theta),\ y = \tan(\theta)$$

Throughout this module, and in fact for the rest of the unit, you will also see $y = \sin(\theta)$, $y = \cos(\theta)$, and $y = \tan(\theta)$ where x is used instead of θ.

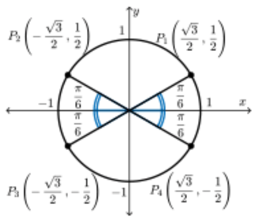
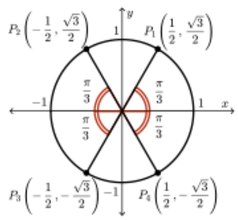
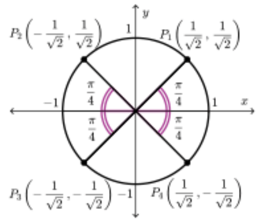
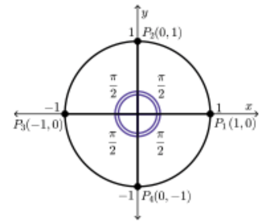
Graphing the Sine Function $y = \sin(x)$



We know the coordinates of points on the unit circle corresponding to standard position angles whose reference angles are $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{2}$ radians. We also know that the y-coordinate of any point on the terminal arm of a standard position angle x and the unit circle is $\sin(x)$.

Graphing the Sine Function $y = \sin(x)$

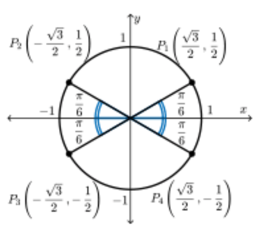
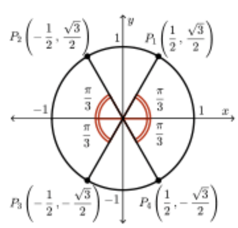
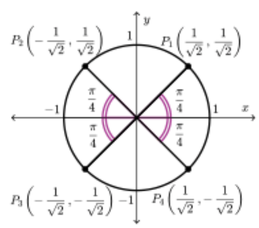
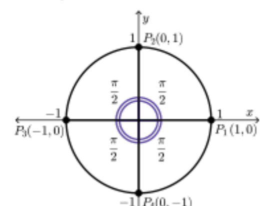
Construct a table of values relating x and $\sin(x)$ for $0 \leq x \leq 2\pi$. Use increments of $\frac{\pi}{12}$ radians (this corresponds to 15° intervals). Fill in the exact values for $\sin(x)$ by referring to the appropriate unit circle diagram. We have not determined the exact values for $\sin(x)$ for which the reference angle is either $\frac{\pi}{12}$ or $\frac{5\pi}{12}$ so we will calculate approximate values later. In the table, these values are shown as "?". Some have been filled in for you.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\sin(x)$		?				?		?	$\frac{\sqrt{3}}{2}$			?	
x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\sin(x)$		?		$-\frac{1}{\sqrt{2}}$		?	-1	?				?	

Graphing the Sine Function $y = \sin(x)$

Construct a table of values relating x and $\sin(x)$ for $0 \leq x \leq 2\pi$. Use increments of $\frac{\pi}{12}$ radians (this corresponds to 15° intervals). Fill in the exact values for $\sin(x)$ by referring to the appropriate unit circle diagram. We have not determined the exact values for $\sin(x)$ for which the reference angle is either $\frac{\pi}{12}$ or $\frac{5\pi}{12}$ so we will calculate approximate values later. In the table, these values are shown as "?". Some have been filled in for you.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\sin(x)$	0	?	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	?	1	?	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	?	0
x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\sin(x)$	0	?	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	?	-1	?	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	?	0

Graphing the Sine Function $y = \sin(x)$

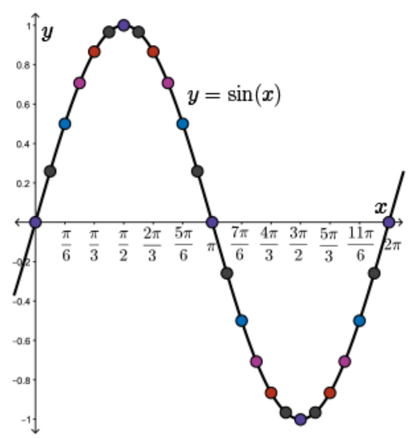
From the table of values, we can plot points to determine the shape of the function $y = \sin(x)$.
The exact values of $\sin(x)$ have been converted to approximate values in the following table, correct to two decimals, where necessary.
A calculator was used to determine the approximate values for multiples of $\frac{\pi}{12}$.
These were marked with a "?" mark in the previous chart.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\sin(x)$	0	0.26	0.5	0.71	0.87	0.97	1	0.97	0.87	0.71	0.5	0.26	0
x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\sin(x)$	0	-0.26	-0.5	-0.71	-0.87	-0.97	-1	-0.97	-0.87	-0.71	-0.5	-0.26	0

Graphing the Sine Function $y = \sin(x)$

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\sin(x)$	0	0.26	0.5	0.71	0.87	0.97	1	0.97	0.87	0.71	0.5	0.26	0
x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\sin(x)$	0	-0.26	-0.5	-0.71	-0.87	-0.97	-1	-0.97	-0.87	-0.71	-0.5	-0.26	0

Plot the points on a graph. The x-axis uses intervals of $\frac{\pi}{12}$ and the y-axis uses intervals of 0.2.
Connect the points on the graph with a smooth curve.
What happens if we continue our table from 2π to 4π?
It is relatively easy to see that the values in the table will repeat since we are passing through the same points on the unit circle each time we complete a full rotation.
Another way to discuss this is through the use of coterminal angles.
For example, $\frac{25\pi}{12}$ is coterminal with $\frac{\pi}{12}$.
On the unit circle, the terminal arm will intersect the unit circle at the same point for both $\frac{25\pi}{12}$ and $\frac{\pi}{12}$.
It follows that $\sin\left(\frac{25\pi}{12}\right) = \sin\left(\frac{\pi}{12}\right)$.



Properties of the Sine Function $y = \sin(x)$

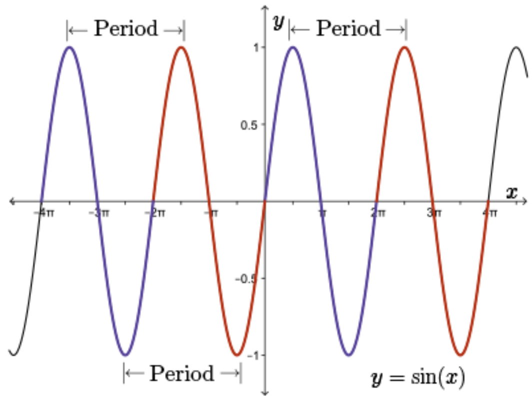
At this point, we will make some observations about the function $y = \sin(x)$ and introduce some new terminology.

The domain of the function $y = \sin(x)$ is $\{x \mid x \in \mathbb{R}\}$.

The maximum value of the sine function is 1 and the minimum value is -1.

Therefore, the range of $y = \sin(x)$ is $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$.
The sine function cycles, that is it repeats over regular intervals of its domain.

We say that sine function is **periodic**.
The horizontal length of one cycle is called the **period**.
The period of $y = \sin(x)$ is 2π or 360°.



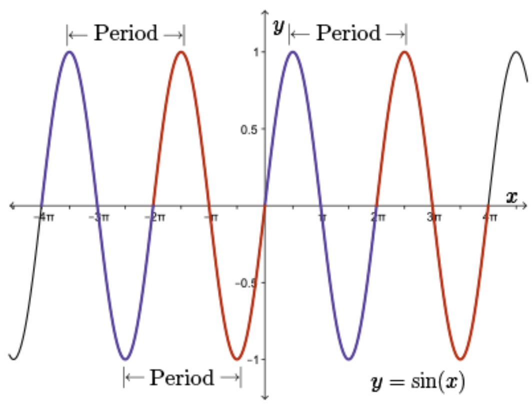
Properties of the Sine Function $y = \sin(x)$

The y-intercept of the sine function is $y = 0$.
There are many x-intercepts. On the graph, the x-intercepts are

$$\{-4\pi, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi\}$$

In general, the x-intercepts are $x = n\pi, n \in \mathbb{Z}$ for x in radians and $x = n(180^\circ), n \in \mathbb{Z}$ for x measured in degrees.

A horizontal central line can be drawn through the sine curve so that the perpendicular distance from this line to a maximum point is the same as the perpendicular distance to a minimum point.
This distance is called the **amplitude**.



In this case, the horizontal central line is $y = 0$, the x-axis, and the amplitude of the sine function is 1.

We used radian measure for the angles. The process of graphing $y = \sin(x)$ does not change if the angle measure is in degrees.

Examples

Example 1

Construct a table of values relating x and $\cos(x)$ for $0 \leq x \leq 2\pi$. Use increments of $\frac{\pi}{12}$ radians (this corresponds to 15° intervals). State exact values for $\cos(x)$ where possible.

Solution

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\cos(x)$	1	$?$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$?$	0	$?$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$?$	-1

x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\cos(x)$	-1	$?$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$?$	0	$?$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$?$	1

Examples

Example 2

Express the exact values from the previous table as decimals, rounded to two decimals where necessary.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\cos(x)$	1	$?$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$?$	0	$?$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$?$	-1

x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\cos(x)$	-1	$?$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$?$	0	$?$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$?$	1

Solution

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\cos(x)$	1	0.97	0.87	0.71	0.5	0.26	0	-0.26	-0.5	-0.71	-0.87	-0.97	-1

x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\cos(x)$	-1	-0.97	-0.87	-0.71	-0.5	-0.26	0	0.26	0.5	0.71	0.87	0.97	1

Examples

Example 3

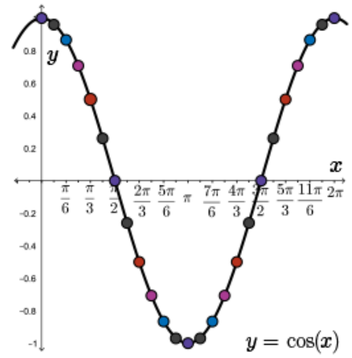
Sketch $y = \cos(x)$ for $0 \leq x \leq 2\pi$. Use increments of $\dfrac{\pi}{12}$ radians (this corresponds to 15° intervals). State the x and y-intercepts, the domain and range, the period, and the amplitude.

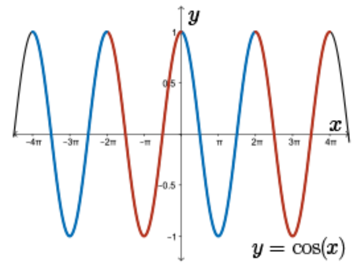
Solution

From the table of values, we obtain the sketch.
A sketch showing more cycles is also shown.

The domain of the function $y = \cos(x)$ is $\{x \mid x \in \mathbb{R}\}$.

The maximum value of the cosine function is 1 and the minimum value is -1.
Therefore, the range of $y = \cos(x)$ is $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$.





Examples

Example 3

Sketch $y = \cos(x)$ for $0 \leq x \leq 2\pi$. Use increments of $\dfrac{\pi}{12}$ radians (this corresponds to 15° intervals). State the x and y-intercepts, the domain and range, the period, and the amplitude.

Solution

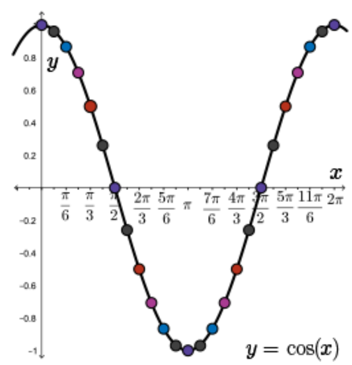
The cosine function has a period of 2π or 360°.
The y-intercept of the cosine function is $y = 1$.

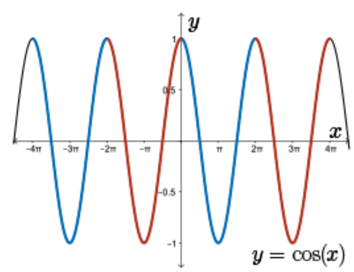
There are many x-intercepts. On the graph, the x-intercepts are

$$\left\{-\frac{7\pi}{2}, -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}\right\}$$

In general, the x-intercepts are $x = \dfrac{\pi}{2} + n\pi, n \in \mathbb{Z}$ for x measured in radians and $x = 90^{\circ} + n(180^{\circ}), n \in \mathbb{Z}$ for x measured in degrees.
A horizontal central line is $y = 0$, the x-axis, and the amplitude of the cosine function is 1.

The sine function and the cosine function are referred to as **sinusoidal curves**. Sinusoidal curves have the property that they oscillate above and below a central horizontal line.





Examples

Example 4

Construct a table of values relating x and $\tan(x)$ for $0 \leq x \leq 2\pi$. Use increments of $\frac{\pi}{12}$ radians (this corresponds to 15° intervals). State the exact values for $\tan(x)$ where possible. Recall that $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

Solution

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\sin(x)$	0	?	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	?	1	?	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	?	0
$\cos(x)$	1	?	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	?	0	?	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	?	-1
$\tan(x)$	0	0.27	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	3.73	undef.	-3.73	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	-0.27	0
x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\sin(x)$	0	?	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	?	-1	?	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	?	0
$\cos(x)$	-1	?	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	?	0	?	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	?	1
$\tan(x)$	0	0.27	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	3.73	undef.	-3.73	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	-0.27	0

Examples

Example 5

Plot the points from the previous table and join them with a smooth curve.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$\sin(x)$	0	?	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	?	1	?	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	?	0
$\cos(x)$	1	?	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	?	0	?	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	?	-1
$\tan(x)$	0	0.27	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	3.73	undef.	-3.73	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	-0.27	0
x	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{17\pi}{12}$	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
$\sin(x)$	0	?	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	?	-1	?	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	?	0
$\cos(x)$	-1	?	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	?	0	?	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	?	1
$\tan(x)$	0	0.27	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	3.73	undef.	-3.73	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	-0.27	0

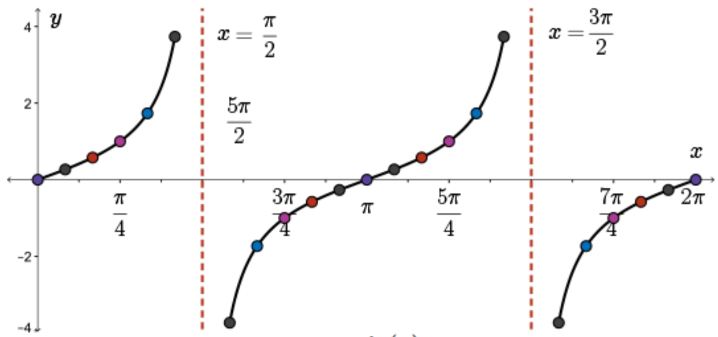
Examples

Example 5

Plot the points from the previous table and join them with a smooth curve.

Solution

Connecting the points with a smooth curve, we obtain the sketch shown.



When $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$, $\cos(x) = 0$ and $\tan(x) = \frac{\sin(x)}{\cos(x)}$ are undefined.

Therefore, there are vertical asymptotes at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. But what happens near the asymptotes? We will check values close to $\frac{\pi}{2}$.

Examples

Example 5

Plot the points from the previous table and join them with a smooth curve.

Solution

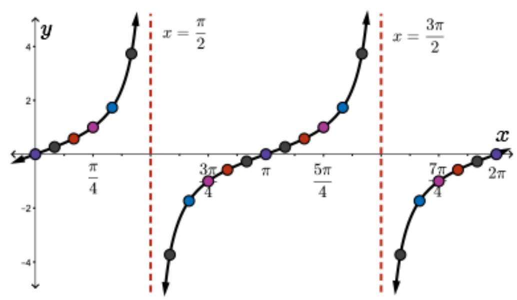
x	Approx. Value of $\tan(x)$
$\frac{\pi}{2} - 0.1$	10.0
$\frac{\pi}{2} - 0.01$	100.0
$\frac{\pi}{2} - 0.001$	1000.0

x	Approx. Value of $\tan(x)$
$\frac{\pi}{2} + 0.1$	-10.0
$\frac{\pi}{2} + 0.01$	-100.0
$\frac{\pi}{2} + 0.001$	-1000.0

As x gets closer to $\frac{\pi}{2}$ from the left, $\tan(x) \to \infty$.

As x gets closer to $\frac{\pi}{2}$ from the right, $\tan(x) \to -\infty$.

We would obtain the same result if we were to check values of x near any of the vertical asymptotes.

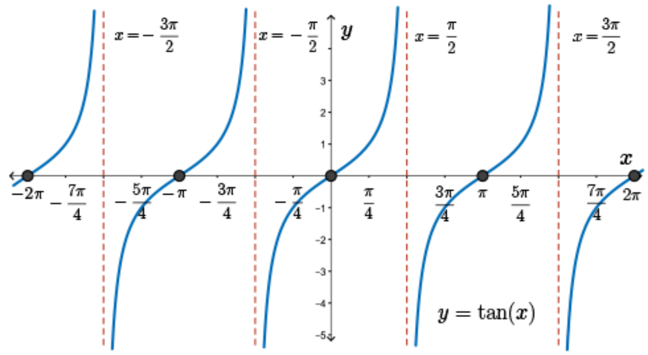


Examples

Example 6

A sketch showing several cycles of the tangent function is shown.
For the function $y = \tan(x)$, state:

- the period and amplitude,
- the equations of any asymptotes,
- the domain and range, and
- the x and y-intercepts



Solution

The function cycles every π radians or every $180°$, so the period of the tangent function is π radians or $180°$.
Since the function has no minimum or maximum value, the amplitude of the tangent function is not defined.

Examples

Example 6

A sketch showing several cycles of the tangent function is shown.
For the function $y = \tan(x)$, state:

- the period and amplitude,
- the equations of any asymptotes,
- the domain and range, and
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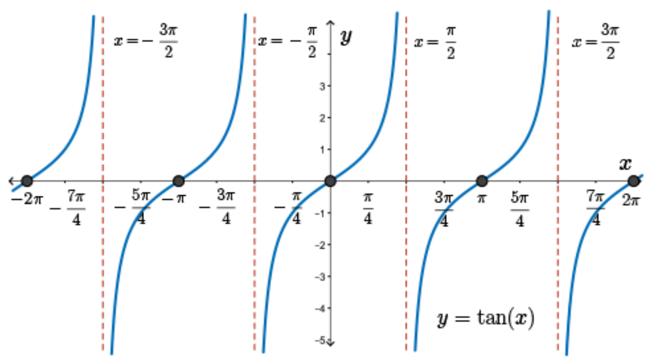


Solution

The graph crosses the x-axis at the origin. The x-intercepts occur every π radians or $180°$.
In general, the x-intercepts occur for $x = n\pi, n \in \mathbb{Z}$ for x measured in radians and $x = n(180°), n \in \mathbb{Z}$ for x measured in degrees. The y-intercept is 0. The vertical asymptotes occur whenever $\cos(x) = 0$.
That is, the vertical asymptotes are $x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$ for x measured in radians and $x = 90° + n(180°), n \in \mathbb{Z}$ for x measured in degrees. There are no horizontal asymptotes.
Since there are vertical asymptotes, it follows that the domain is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\right\}$ for x measured in radians and $\{x \mid x \neq 90° + n(180°), n \in \mathbb{Z}, x \in \mathbb{R}\}$ for x measured in degrees.
The range is $\{y \mid y \in \mathbb{R}\}$.

Examples

Example 7

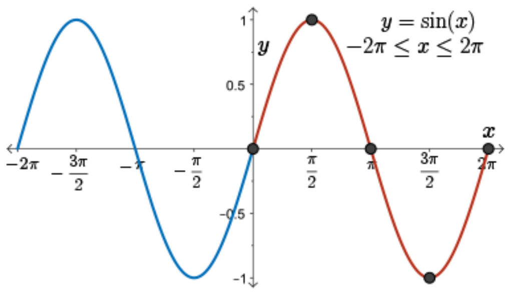
Using the y-intercept, the x-intercepts, and the maximum and minimum values, sketch $y = \sin(x)$ and $y = \cos(x)$ for $-2\pi \le x \le 2\pi$ and for $-360^\circ \le x \le 360^\circ$.

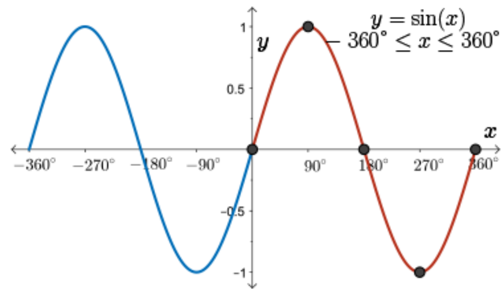
Solution

For $y = \sin(x)$, you can identify five key points to sketch one period.

In radian measure, these points are $(0, 0)$, $\left(\frac{\pi}{2}, 1\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -1\right)$, and $(2\pi, 0)$.

From there, you can sketch as many periods as you require.





Examples

Example 8

Using three key points and the vertical asymptotes, sketch $y = \tan(x)$ for $-2\pi \le x \le 2\pi$ and for $-360^\circ \le x \le 360^\circ$.

Solution

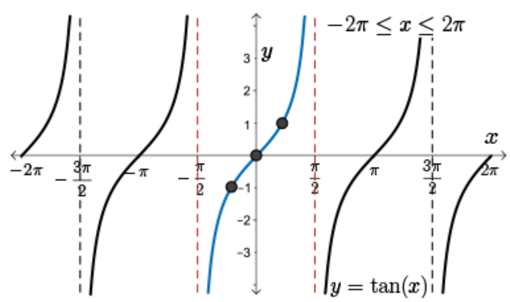
For $y = \tan(x)$, we know that there are vertical asymptotes located at $x = \pm\frac{\pi}{2}$.

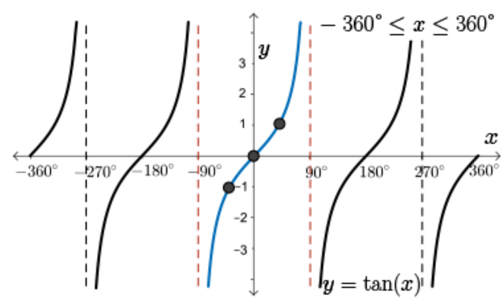
We know the coordinates of points between the asymptotes whose y-coordinates are -1, 0, and 1.

In radian measure, these points are $\left(-\frac{\pi}{4}, -1\right)$, $(0, 0)$, and $\left(\frac{\pi}{4}, 1\right)$.

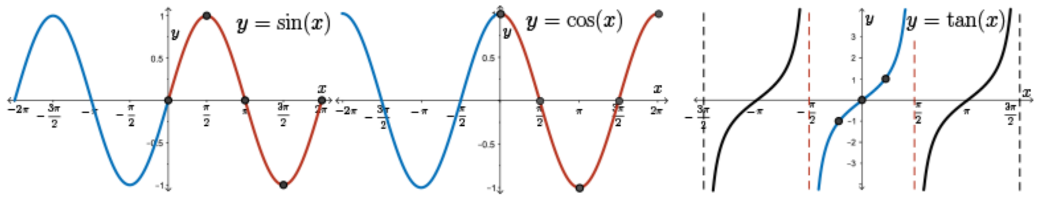
In degree measure, these points are $(-45^\circ, -1)$, $(0^\circ, 0)$, and $(45^\circ, 1)$.

We can efficiently sketch one period. From there, we can sketch as many periods as we like.





Summary



The results are summarized for radian measure in the following table.

	$y = \sin(x)$	$y = \cos(x)$	$y = \tan(x)$
Domain	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$	$\left\{x \mid x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\right\}$
Range	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$
Maximum	$y = 1$	$y = 1$	none
Minimum	$y = -1$	$y = -1$	none
Period	2π	2π	π
Amplitude	1	1	not defined
Vertical Asymptotes	none	none	$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
y-intercept	0	1	0
x-intercepts	$x = n\pi, \in \mathbb{Z}$	$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$	$x = n\pi, n \in \mathbb{Z}$

In a future module, we will sketch 3 reciprocal trigonometric functions: $y = \csc(x)$, $y = \sec(x)$, and $y = \cot(x)$.