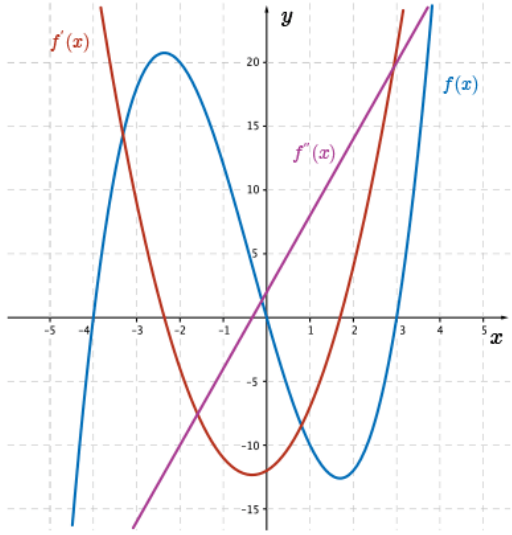
Sketching the First and Second Derivative Functions

Sketching Graphs of Derivative Functions

Previously, we have seen that if $f(x)$ is a polynomial of degree n, then its derivative is one degree lower (i.e., $n-1$). (One exception to this is the case where $f(x)$ is a constant function and so has degree $n=0$.)

For example, if $f(x)$ is a cubic polynomial (degree 3), then its derivative function is quadratic (degree 2). The derivative of the quadratic function is linear, so the second derivative function of a cubic polynomial is linear (degree 1).



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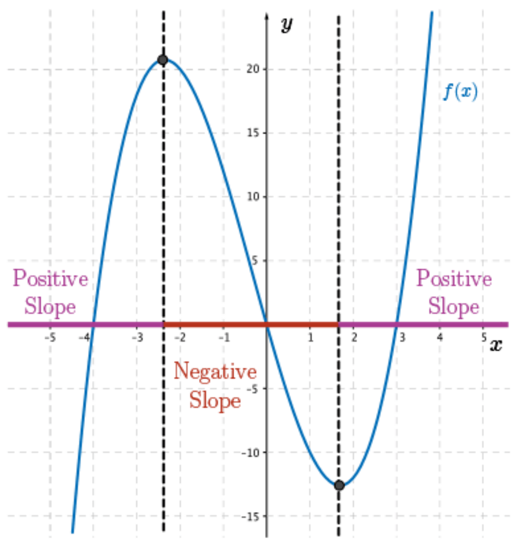
Also, as we move from the left side to the right side of the graph of a polynomial with degree $n \geq 2$, we notice that the slope of the tangent line changes in steepness and over certain intervals the slope is positive or negative.



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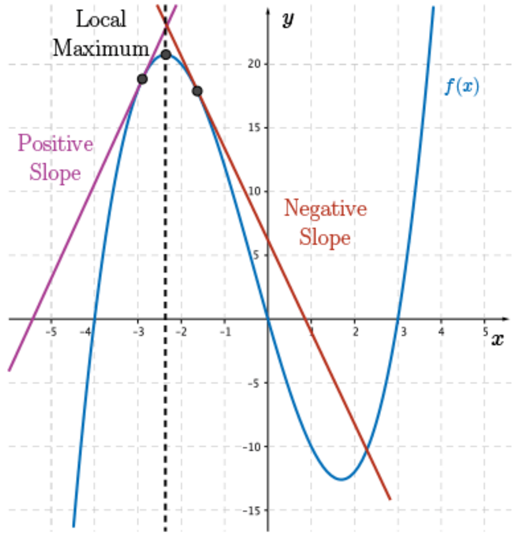
Also, as we move from the left side to the right side of the graph of a polynomial with degree $n \geq 2$, we notice that the slope of the tangent line changes in steepness and over certain intervals the slope is positive or negative.

The slope of the tangent line changes from positive to negative as we pass through a local maximum.



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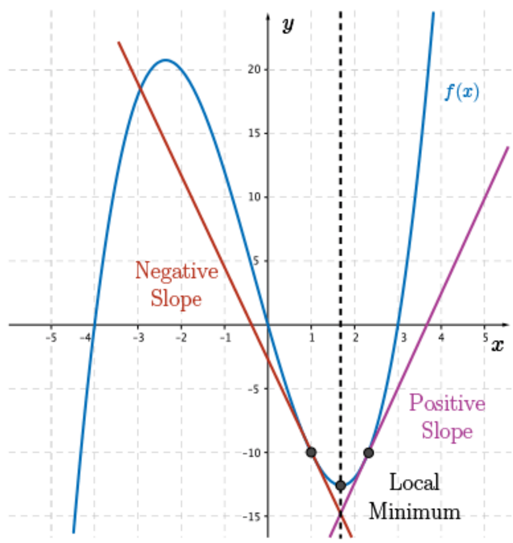
Also, as we move from the left side to the right side of the graph of a polynomial with degree $n \geq 2$, we notice that the slope of the tangent line changes in steepness and over certain intervals the slope is positive or negative.

The slope of the tangent line changes from positive to negative as we pass through a local maximum.

The slope of the tangent line changes from negative to positive as we pass through a local minimum.



Sketching Graphs of Derivative Functions

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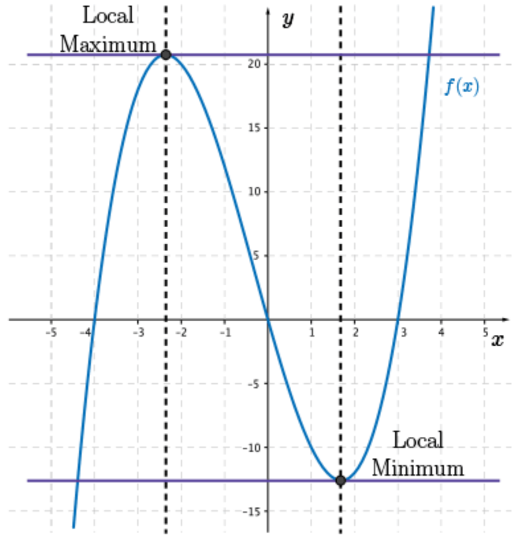
For example, if $f(x)$ is a cubic polynomial (degree 3), then its derivative function is quadratic (degree 2). The derivative of the quadratic function is linear, so the second derivative function of a cubic polynomial is linear (degree 1).

Also, as we move from the left side to the right side of the graph of a polynomial with degree $n \geq 2$, we notice that the slope of the tangent line changes in steepness and over certain intervals the slope is positive or negative.

The slope of the tangent line changes from positive to negative as we pass through a local maximum.

The slope of the tangent line changes from negative to positive as we pass through a local minimum.

At the local maximum or minimum, we see that the tangent line is horizontal and therefore, the value of its derivative is 0.



We can use all of this information to help us sketch the graph of the derivative function.

Examples

Example 1

Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

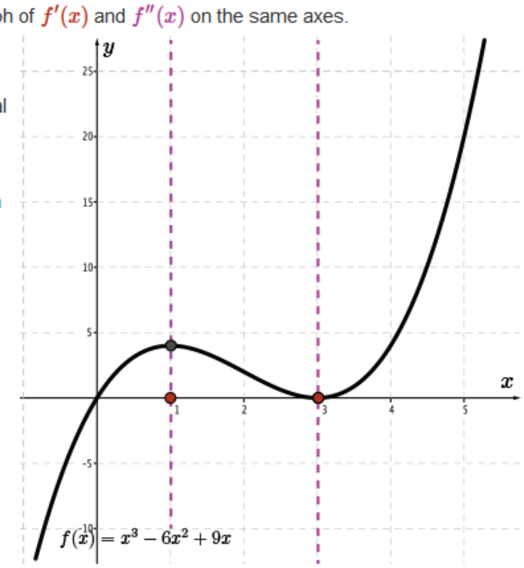
To sketch $f'(x)$, begin by identifying the position of the local maximum and local minimum.

Through these points, draw vertical lines to separate the intervals where the slope of the tangent line is positive from the interval where the slope of the tangent line is negative.

Since the slope of the tangent line is 0 at the local maximum and minimum, at these same x-coordinates the value of $f'(x) = 0$.

So, we know two points on the graph of $f'(x)$: $(1, 0)$ and $(3, 0)$.

Notice how the positions of the local maximums and minimums of the graph of $f(x)$ become the positions for the x-intercepts or zeros of the graph of $f'(x)$.



Examples

Example 1

Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

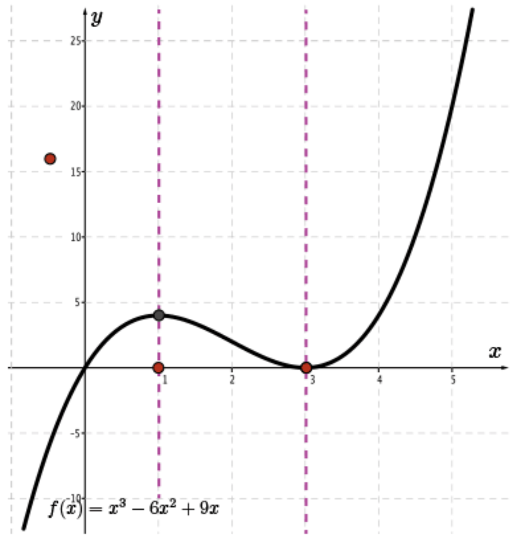
Given that $f(x)$ is a cubic function, $f'(x)$ must be quadratic.

In the left-most interval, $x < 1$, the slope of the tangent line at any point within this interval will always be positive. Therefore, we know that $f'(x) > 0$ within this interval.

Let's examine the tangent drawn to the point where $x = -0.5$.

Counting the rise and run, we find that the slope of the tangent at this point has a slope value of about $m = \frac{8}{0.5} = 16$.

Therefore, we will use the point $(-0.5, 16)$ to help sketch the graph of $f'(x)$.



Examples

Example 1

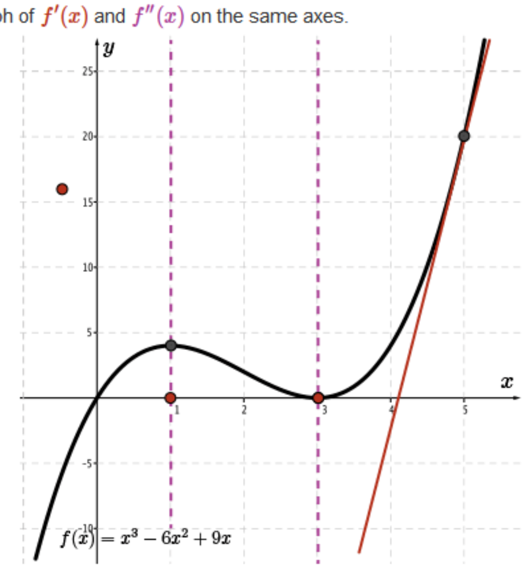
Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

Similarly, in the right-most interval, the slope of the tangent line at any point within this interval will always be positive. Therefore, we know that $f'(x) > 0$ within this interval.

Let's examine the tangent drawn to the point where $x = 5$.

Counting the rise and run, we find that the slope of the tangent at this point has a slope value of about $m = \frac{24}{1} = 24$.



Examples

Example 1

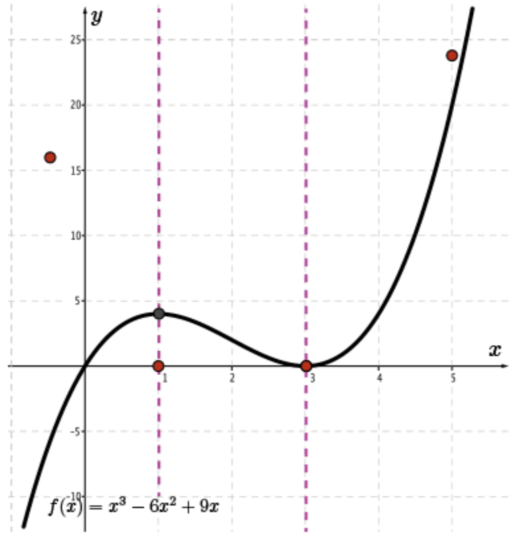
Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

Similarly, in the right-most interval, the slope of the tangent line at any point within this interval will always be positive. Therefore, we know that $f'(x) > 0$ within this interval.

Let's examine the tangent drawn to the point where $x = 5$. Counting the rise and run, we find that the slope of the tangent at this point has a slope value of about $m = \frac{24}{1} = 24$.

Therefore, we will use the point $(5, 24)$ to help sketch the graph of $f'(x)$.



Examples

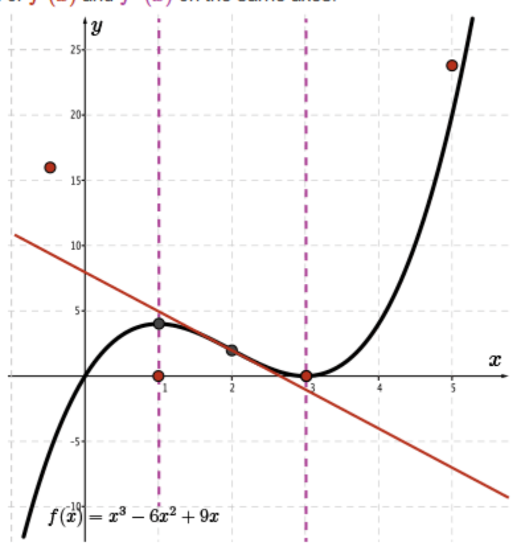
Example 1

Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

Within the middle interval, the slope of the tangent to the curve will always be negative, meaning that $f'(x) < 0$. Since we know that $f'(x)$ is quadratic and we know the position of its two zeros, there is a minimum value that will occur on $f'(x)$ halfway between the two zeros. For this graph, the minimum value will occur at $x = 2$.

Counting the rise and run, we find that the slope of the tangent at the point where $x = 2$ has a slope value of about $m = \frac{-6}{2} = -3$.



Examples

Example 1

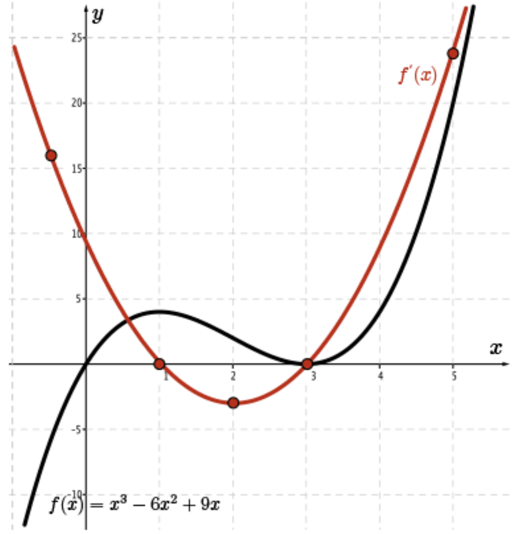
Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

Within the middle interval, the slope of the tangent to the curve will always be negative, meaning that $f'(x) < 0$.
Since we know that $f'(x)$ is quadratic and we know the position of its two zeros, there is a minimum value that will occur on $f'(x)$ halfway between the two zeros. For this graph, the minimum value will occur at $x = 2$.

Counting the rise and run, we find that the slope of the tangent at the point where $x = 2$ has a slope value of about $m = \frac{-6}{2} = -3$.
Therefore, the point $(2, -3)$ is on the sketch of $f'(x)$.

We now have five points on the graph of $f'(x)$ and we know that this derivative function is quadratic.
We connect these points with a sweeping parabolic curve.



Examples

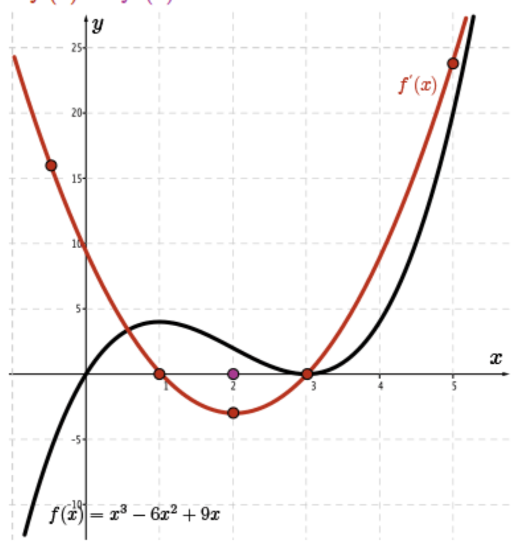
Example 1

Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

To sketch the graph of $f''(x)$, the same reasoning process is applied to the graph of $f'(x)$.
Since $f'(x)$ is quadratic, $f''(x)$ is linear.
There is only one minimum: the vertex of the parabola. At this point, we know that the value of the derivative is 0.

Therefore, $f''(2) = 0$, and we know that the point $(2, 0)$ is on the sketch of $f''(x)$.



Examples

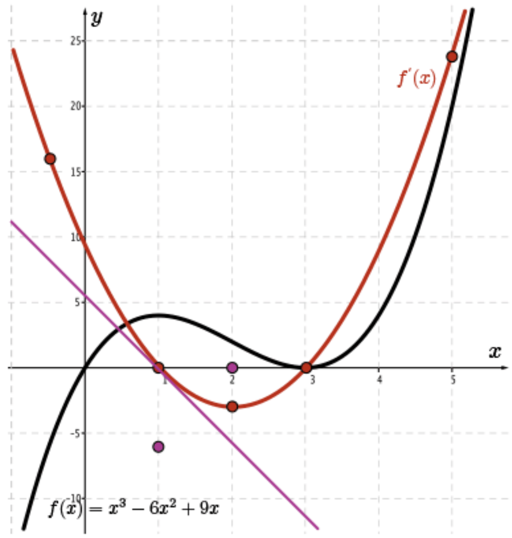
Example 1

Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

To the left of the vertex, $f'(x)$ is decreasing and the tangent line has a negative slope, so we know that $f''(x) < 0$.

Let's calculate the slope of a tangent drawn to the sketch of $f'(x)$ at the point where $x = 1$.
Counting the rise and run, we find that the slope of the tangent at the point where $x = 1$ has a slope value of about $m = \frac{-6}{1} = -6$.
Therefore, the point $(1, -6)$ is on the sketch of $f''(x)$.



Examples

Example 1

Using the graph of $f(x) = x^3 - 6x^2 + 9x$, sketch the graph of $f'(x)$ and $f''(x)$ on the same axes.

Solution

To the right of the vertex, $f'(x)$ is increasing and the tangent line has a positive slope. In other words, $f''(x) > 0$.
Since $f'(x)$ is quadratic and therefore symmetrical about its axis, the slope of the tangent at $x = 1$ and at $x = 3$ will have the same steepness but will be opposite in direction.
Since the slope of the tangent at $x = 1$ is -6, the slope of the tangent at $x = 3$ is $+6$.
Therefore, the point $(3, 6)$ is on the sketch of $f''(x)$.

We have three points on the sketch of $f''(x)$ and we know that the second derivative function is linear.
Therefore, we connect these points to make the graph, as shown.